

Hawking Radiation from Non-Extremal D1-D5 Black Hole via Anomalies

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Abstract

We take the method of anomaly cancellation for the derivation of Hawking radiation initiated by Robinson and Wilczek, and apply it to the non-extremal five-dimensional D1-D5 black hole in string theory. The fluxes of the electric charge flow and the energy-momentum tensor from the black hole are obtained. They are shown to match exactly with those of the two-dimensional black body radiation at the Hawking temperature.

Keywords : Hawking radiation, anomaly, non-extremal D1-D5 black hole

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1 Introduction

Hawking radiation first observed by Hawking [1] is the quantum effect of fields in a classical space-time background with an event horizon. Although it is semi-classical, it provides an important key to understand the nature of black hole horizon. Since the quantum effect of gravity itself becomes no longer negligible near the black hole horizon, Hawking radiation also provides a basic information in formulating the theory of quantum gravity, like the string theory which is one of the candidates.

Usually, a certain physical result may have various mathematical formulations for obtaining it and interpretations from various different angles. Having various viewpoints is always useful and important in deepening the understanding of it. As for Hawking radiation, a new interpretation has been proposed by Robinson and Wilczek [2]. They have shown that the Hawking radiation plays the role of preserving general covariance at the quantum level by canceling the diffeomorphism anomaly at the event horizon. Actually, there is a similar work [3] that also considers the Hawking radiation from the viewpoint of anomaly. However, as noted in [2], it is specialized to two-dimensional space-time. On the other hand, the derivation of Hawking radiation based on anomaly cancellation at the horizon does not depend on the space-time dimension, and confirms that Hawking radiation is a universal phenomenon.

The proposal by Robinson and Wilczek, which is based on the static and spherically symmetric black hole, has been elaborated in [4, 5] where, via extensions to charged and rotating black holes, it has been shown that Hawking radiation is capable of canceling anomalies of local symmetries at the horizon. After this elaboration, there have been many subsequent works which apply the method of anomaly cancellation to various black holes in various dimensions and verify the validity of the method [6]- [13]. Further investigation on the derivation of Hawking flux itself has been also given in [14]. In this paper, we give one more example supporting and confirming the method of anomaly cancellation by considering a typical black hole background in string theory. It is expected that our result strengthens the validity and power of the method.

The black hole background we are concerned about is the charged non-extremal five-dimensional black hole in string theory, which is obtained from a specific D-brane configuration and often called the non-extremal D1-D5 black hole [15].¹ This background is particularly interesting since, as noted in [15], it is related to various black solutions by taking different limits on parameters appearing in the background; five-dimensional Reissner-

¹For detailed and comprehensive review on non-extremal D1-D5 black hole, see for example [16].

Nordström and Schwarzschild solutions, six-dimensional black string solution [17], black five-brane solution [18], dyonic black string solution [19]. So it may be argued that Hawking radiation from several black backgrounds can be discussed by considering just one background.

The organization of this paper is as follows: After a brief description on the non-extremal five-dimensional D1-D5 black hole from the in the next section, we consider a test charged scalar field in the black hole background in Sec. 3, and show that, near the horizon, the action for the scalar field reduces to a two-dimensional theory in a certain background. In Sec. 4, we calculate the fluxes of the electric charge flows and the energy-momentum tensor by applying the method of anomaly cancellation to the effective two-dimensional theory, and show that the results match exactly with the fluxes of black body radiation at Hawking temperature. Finally, the discussion follows in Sec. 5.

2 Non-extremal five-dimensional D1-D5 black hole

The non-extremal five-dimensional black hole originates from a brane configuration in Type IIB superstring theory compactified on $S^1 \times T^4$. The configuration relevant to the present case is composed of D1-branes wrapping S^1 , D5-branes wrapping $S^1 \times T^4$, and momentum modes along S^1 . The solution of the Type IIB supergravity corresponding to this configuration is a supersymmetric background known as the extremal five-dimensional D1-D5 black hole. The extremal black hole preserves some fraction of supersymmetry and hence has zero Hawking temperature, which implies that we do not see Hawking radiation. Therefore, in order to consider the Hawking radiation, we need the non-extremal version of the extremal solution.

Let x_5 and x_6, \dots, x_9 be periodic coordinates along S^1 and T^4 , respectively. Then the ten-dimensional supergravity background corresponding to the non-extremal D1-D5 black hole has the following form in the string frame [15]:

$$\begin{aligned} ds_{10}^2 &= f_1^{-1/2} f_5^{-1/2} (-h f_n^{-1} dt^2 + f_n (dx_5 + (1 - \tilde{f}_n^{-1}) dt)^2) \\ &\quad + f_1^{1/2} f_5^{-1/2} (dx_6^2 + \dots + dx_9^2) + f_1^{1/2} f_5^{1/2} (h^{-1} dr^2 + r^2 d\Omega_3^2) , \\ e^{-2\phi} &= f_1^{-1} f_5 , \quad C_{05} = \tilde{f}_1^{-1} - 1 , \\ F_{ijk} &= \frac{1}{2} \epsilon_{ijkl} \partial_l \tilde{f}_5 , \quad i, j, k, l = 1, 2, 3, 4 , \end{aligned} \tag{2.1}$$

where F is the three-form field strength of the RR 2-form gauge potential C , $F = dC$. Various

functions appearing in the background are functions of coordinates x_1, \dots, x_4 given by

$$\begin{aligned} h &= 1 - \frac{r_0^2}{r^2}, \quad f_{1,5,n} = 1 + \frac{r_{1,5,n}^2}{r^2}, \\ \tilde{f}_{1,n}^{-1} &= 1 - \frac{r_0^2 \sinh \alpha_{1,n} \cosh \alpha_{1,n}}{r^2} f_{1,n}^{-1}, \\ r_{1,5,n}^2 &= r_0^2 \sinh^2 \alpha_{1,5,n}, \quad r^2 = x_1^2 + \dots + x_4^2, \end{aligned} \quad (2.2)$$

where r_0 is the extremality parameter. Here, h and $f_{1,5,n}$, are harmonic functions representing the non-extremality and the presence of D1, D5, and momentum modes, respectively.

Upon dimensional reduction of Eq. (2.1) along $S^1 \times T^4$ following the procedure of [20], we get the Einstein metric of the non-extremal five-dimensional black hole as

$$ds_5^2 = -\lambda^{-2/3} h dt^2 + \lambda^{1/3} (h^{-1} dr^2 + r^2 d\Omega_3^2), \quad (2.3)$$

where λ is defined by

$$\lambda = f_1 f_5 f_n. \quad (2.4)$$

The location of the event horizon, r_H , of this black hole geometry is obtained as

$$r_H = r_0. \quad (2.5)$$

Apart from the metric, the dimensional reduction gives us three kinds of gauge fields. The first one is the Kaluza-Klein gauge field $A_\mu^{(K)}$ coming from the metric, and the second one, say $A_\mu^{(1)}$, basically stems from $C_{\mu 5}$. (We note that $\mu = 0, 1, 2, 3, 4$.) From the background of Eq. (2.1), two gauge fields are obtained as

$$A^{(K)} = -(\tilde{f}_n^{-1} - 1)dt, \quad A^{(1)} = (\tilde{f}_1^{-1} - 1)dt. \quad (2.6)$$

Unlike these gauge fields which are one-form in nature, the last one is the two-form gauge field, $A_{\mu\nu}$, originating from $C_{\mu\nu}$, whose field strength is given by the expression of F in Eq. (2.1). Though this two-form gauge field gives a non-zero contribution to the full black hole background, it will not play any role in the remaining part of this paper, and thus be excluded in our consideration from now on. Then, the background composed of Eqs. (2.3) and (2.6) will be our concern.

3 Quantum field near the horizon

In this section, we consider a free complex scalar field in the black hole background, Eqs. (2.3) and (2.6), and investigate its action near the horizon based on the observation of Ref. [2].

The field is taken to have minimal coupling to the gauge fields, Eq. (2.6). We would like to note that this gives a simple reason why the two-form gauge field does not enter seriously in our study; the object minimally coupled to the two-form gauge field is not point-like but string-like one.

The action for the complex scalar field φ in the background, Eqs. (2.3) and (2.6), is evaluated as

$$\begin{aligned} S[\varphi] &= - \int d^5x \sqrt{-g} g^{\mu\nu} (D_\mu \varphi)^* D_\nu \varphi \\ &= - \int dt dr r^3 \int d\Omega_3 \varphi^* \left(-\frac{\lambda}{h} D_t^2 + \frac{1}{r^3} \partial_r r^3 h \partial_r + \frac{1}{r^2} \nabla_\Omega^2 \right) \varphi , \end{aligned} \quad (3.1)$$

where $\int d\Omega_3$ and ∇_Ω^2 denote the integration and the Laplacian on unit three sphere, respectively, and $D_t = \partial_t - ie_1 A_t^{(1)} - ie_K A_t^{(K)}$ with the $U(1)$ charges e_1 and e_K is the covariant derivative.

First of all, we perform the partial wave decomposition of φ in terms of the spherical harmonics on S^3 as $\varphi = \sum_a \varphi_a Y_a$, where a is the collection of angular quantum numbers of the spherical harmonics and φ_a depends on the coordinates, t and r . Then we see that the action is reduced to a two-dimensional effective theory with an infinite collection of fields labeled by a . Next, in order to see what happens near the horizon, it is helpful to take a transformation to the tortoise coordinate r^* , which, in our case, is defined by

$$\frac{\partial r^*}{\partial r} = \frac{\lambda^{1/2}}{h} \equiv \frac{1}{f(r)} , \quad (3.2)$$

and leads to $\int dr = \int dr^* f(r(r^*))$. In the region near the horizon, $f(r(r^*))$ (or $h(r(r^*))$) appears to be a suppression factor vanishing exponentially fast, and thus the terms in the action which do not have some factor compensating it can be ignored. In our case, the terms coming from the Laplacian on unit three sphere are suppressed by $f(r(r^*))$. We note that the suppression also takes place for the mass term or the interaction terms of φ when they are included in the action (3.1). Therefore, quite generically, the action near the horizon becomes

$$S[\varphi] = - \sum_a \int dt dr r^3 \lambda^{1/2} \varphi_a^* \left(-\frac{1}{f} (\partial_t - iA_t)^2 + \partial_r f \partial_r \right) \varphi_a , \quad (3.3)$$

where $A_t = e_1 A_t^{(1)} + e_K A_t^{(K)}$. Now it is not hard to find that this action describes an infinite set of massless two-dimensional complex scalar fields in the following background:

$$\begin{aligned} ds^2 &= -f(r) dt^2 + \frac{1}{f(r)} dr^2 , \quad \Phi = r^3 \lambda^{1/2} , \\ A_t &= -\frac{e_1 r_0^2 \sinh \alpha_1 \cosh \alpha_1}{r^2 + r_1^2} + \frac{e_K r_0^2 \sinh \alpha_n \cosh \alpha_n}{r^2 + r_n^2} , \end{aligned} \quad (3.4)$$

where Φ is the two-dimensional dilaton field.

What we have seen is that the physics near the horizon of the original five-dimensional theory (3.1) is effectively described by a two-dimensional theory, which is non-interacting and massless one (3.3).

4 Anomalies and Hawking fluxes

Having the two-dimensional effective field theory near the horizon (3.3), we consider the problem of Hawking radiation following the approach based on the anomaly cancellation proposed in [2, 4].

One important ingredient of the anomaly approach of [2] is to notice that, since the horizon is a null hypersurface, all ingoing (left moving) modes at the horizon can not classically affect physics outside the horizon. This implies that they may be taken to be out of concern at the classical level and thus the effective two-dimensional theory becomes chiral, that is, the theory only of outgoing (right moving) modes. If we now perform the path integration of right moving modes, the resulting quantum effective action becomes anomalous under the gauge or the general coordinate transformation, due to the absence of the left moving modes. However, such anomalous behaviors are in contradiction to the fact that the underlying theory is not anomalous. The reason for this is simply that we have ignored the quantum effects of the classically irrelevant left moving modes at the horizon. Thus anomalies must be cancelled by including them. In what follows, anomaly cancellations at the horizon are studied and their relation to the Hawking fluxes is investigated.

The previous paragraph states that anomalies appear at the horizon r_H . For computational convenience, we regard the quantum effective action to be anomalous in an infinitesimal slab, $r_H \leq r \leq r_H + \epsilon$, which is the region near the horizon. (The limit $\epsilon \rightarrow 0$ is taken at the end of the calculation.) This leads to a splitting of the region outside the horizon, $r_H \leq r \leq \infty$, into two regions, $r_H \leq r \leq r_H + \epsilon$ and $r_H + \epsilon \leq r \leq \infty$. Then, since the field we are considering is charged one, there will be the gauge and the gravitational anomaly near the horizon, $r_H \leq r \leq r_H + \epsilon$.

We first consider the gauge anomaly. Since there are two kinds of $U(1)$ gauge symmetries, we have two $U(1)$ gauge currents, which are denoted as $J_\mu^{(1)}$ and $J_\mu^{(K)}$ following the notation of the original gauge potentials $A_\mu^{(1)}$ and $A_\mu^{(K)}$. The two-dimensional anomalies for these two current are identical in structure. So we will concentrate on the anomaly for $J_\mu^{(1)}$ and give just the result for another current.

Since the region outside the horizon has been divided into two regions, it is natural to

write the gauge current as a sum

$$J^{(1)\mu} = J_{(o)}^{(1)\mu} \Theta_+(r) + J_{(H)}^{(1)\mu} H(r) , \quad (4.1)$$

where $\Theta_+(r) = \Theta(r - r_+ - \epsilon)$ and $H(r) = 1 - \Theta_+(r)$. Apart from the near horizon region, the current is conserved

$$\partial_r J_{(o)}^{(1)r} = 0 . \quad (4.2)$$

On the other hand, the current near the horizon is anomalous and obeys the anomalous equation

$$\partial_r J_{(H)}^{(1)r} = \frac{e_1}{4\pi} \partial_r A_t , \quad (4.3)$$

which is the form of two-dimensional consistent gauge anomaly [21, 22]. Since these two equations in each region are first order differential ones, they can be easily integrated as

$$\begin{aligned} J_{(o)}^{(1)r} &= c_o^{(1)} , \\ J_{(H)}^{(1)r} &= c_H^{(1)} + \frac{e_1}{4\pi} (A_t(r) - A_t(r_H)) , \end{aligned} \quad (4.4)$$

where $c_o^{(1)}$ and $c_H^{(1)}$ are integration constants. We note that $c_o^{(1)}$ is the electric charge flux which we are going to obtain.

Now, we let W be the quantum effective action of the theory without including the ingoing (left moving) modes near the horizon. Then its variation under a gauge transformation with gauge parameter ζ is given by

$$\begin{aligned} -\delta W &= \int d^2x \sqrt{-g} \zeta \nabla_\mu J^{(1)\mu} \\ &= \int d^2x \zeta \left[\partial_r \left(\frac{e_1}{4\pi} A_t H \right) + \delta(r - r_+ - \epsilon) \left(J_{(o)}^{(1)r} - J_{(H)}^{(1)r} + \frac{e_1}{4\pi} A_t \right) \right] , \end{aligned} \quad (4.5)$$

where Eqs. (4.1), (4.2), and (4.3) have been used for obtaining the second line. As alluded to in the early part of this section, the full quantum effective action of the underlying theory must have gauge invariance. The full effective action includes the quantum effects of the ingoing modes near the horizon, whose gauge variation gives a term canceling the first term of (4.5). For the gauge invariance, the coefficient of the delta function in Eq. (4.5) should also vanish, and hence, by using Eq. (4.4), we get

$$c_o^{(1)} = c_H^{(1)} - \frac{e_1}{4\pi} A_t(r_H) . \quad (4.6)$$

In order to determine the charge flux $c_o^{(1)}$, the value of the current at the horizon, $c_H^{(1)}$, should be fixed. This is done by imposing a condition that the covariant current [22] given

by $\tilde{J}^{(1)r} = J^{(1)r} + \frac{e_1}{4\pi} A_t(r) H(r)$ vanishes at the horizon, which, as noted in [5], assures the regularity of physical quantities at the future horizon. Then, the electric charge flux canceling gauge anomaly is determined as

$$c_o^{(1)} = -\frac{e_1}{2\pi} A_t(r_H) = \frac{e_1}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) . \quad (4.7)$$

As for the current $J_\mu^{(K)}$ associated with another $U(1)$ gauge symmetry, we can follow the same steps from Eq. (4.1) to Eq. (4.7), with the anomaly equation

$$\partial_r J_{(H)}^{(K)r} = \frac{e_K}{4\pi} \partial_r A_t , \quad (4.8)$$

and obtain

$$c_o^{(K)} = -\frac{e_K}{2\pi} A_t(r_H) = \frac{e_K}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) . \quad (4.9)$$

As we will see, the electric charge fluxes, (4.7) and (4.9), exactly match with those of the two-dimensional Hawking (blackbody) radiation with the Planck distribution including chemical potentials.

We now turn to the problem of determining the flux of the energy-momentum tensor through the cancellation of the gravitational anomaly. The method for solving it is the same with that adopted in the case of gauge anomaly. First of all, like the splitting of Eq. (4.1), we write the energy-momentum tensor as

$$T_\nu^\mu = T_{\nu(o)}^\mu \Theta_+(r) + T_{\nu(H)}^\mu H(r) . \quad (4.10)$$

Due to the presence of the gauge potentials and the dilaton in the background (3.4), the energy-momentum tensor satisfies the modified conservation equation [4]. What is of interest for our problem is the conservation equation for the component T_t^r , the energy-momentum flux in the radial direction. Apart from the near horizon region, it is given by

$$\partial_r T_{t(o)}^r = J_{(o)}^r \partial_r A_t . \quad (4.11)$$

Here $J_{(o)}^r$ comes from the current $J^r \equiv \frac{1}{e_1} J^{(1)r} = \frac{1}{e_K} J^{(K)r}$ in a splitting like Eq. (4.1) and satisfies $\partial_r J_{(o)}^r = 0$, whose solution is $J_{(o)}^r = c_0$ with $c_0 = \frac{1}{e_1} c_o^{(1)}$ or $\frac{1}{e_K} c_o^{(K)}$. In the near horizon region, we have anomalous conservation equation [4] as

$$\partial_r T_{t(H)}^r = J_{(H)}^r \partial_r A_t + A_t \partial_r J_{(H)}^r + \partial_r N_t^r , \quad (4.12)$$

where $N_t^r = (f'^2 + f f'')/192\pi$. (The prime denotes the derivative with respect to r .) The second term comes from the gauge anomaly represented by the anomalous conservation

equation $\partial_r J_{(H)}^r = \frac{1}{4\pi} \partial_r A_t$, while the third term is due to the gravitational anomaly for the consistent energy-momentum tensor [23]. Now it is not a difficult task to integrate Eqs. (4.11) and (4.12) and obtain

$$\begin{aligned} T_{t(o)}^r &= a_o + c_o A_t , \\ T_{t(H)}^r &= a_H + \int_{r_H}^r dr \partial_r \left(c_o A_t + \frac{1}{4\pi} A_t^2 + N_t^r \right) , \end{aligned} \quad (4.13)$$

where a_o and a_H are integration constants. Here a_o is the energy flux which we are interested in.

Next, we consider the variation of quantum effective action W under a general coordinate transformation in the time direction with a transformation parameter ξ^t :

$$\begin{aligned} -\delta W &= \int d^2x \sqrt{-g} \xi^t \nabla_\mu T_t^\mu \\ &= \int d^2x \xi^t \left[c_o \partial_r A_t + \partial_r \left[\left(\frac{1}{4\pi} A_t^2 + N_t^r \right) H \right] \right. \\ &\quad \left. + \left(T_{t(o)}^r - T_{t(H)}^r + \frac{1}{4\pi} A_t^2 + N_t^r \right) \delta(r - r_+ - \epsilon) \right] . \end{aligned} \quad (4.14)$$

The first term in the second line is purely the classical effect of the background electric field for constant current flow. The second term is cancelled by including the quantum effect of the ingoing modes as is the case of gauge anomaly. The last term gives non-vanishing contribution at the horizon and is also required to vanish for the general covariance of the full quantum effective action. This requirement leads us to have the following relation.

$$a_o = a_H + \frac{1}{4\pi} A_t^2(r_H) - N_t^r(r_H) , \quad (4.15)$$

where the solution Eq. (4.13) has been used. For determining a_o , we first need to know the value of a_H , which is fixed by imposing a condition that the covariant energy-momentum tensor vanishes at the horizon for regularity at the future horizon [5]. Then, from the expression of the covariant energy-momentum tensor [22, 24], $\tilde{T}_t^r = T_t^r + \frac{1}{192\pi} (f f'' - 2(f')^2)$, the condition $\tilde{T}_t^r(r_H) = 0$ gives

$$a_H = \frac{\kappa^2}{24\pi} = 2N_t^r(r_H) , \quad (4.16)$$

where κ is the surface gravity at the horizon,

$$\kappa = 2\pi T_H = \frac{1}{2} \partial_r f|_{r=r_H} = \frac{1}{r_0 \cosh \alpha_1 \cosh \alpha_5 \cosh \alpha_n} . \quad (4.17)$$

Here we see that the Hawking temperature of the non-extremal D1-D5 black hole is

$$T_H = \frac{1}{2\pi r_0 \cosh \alpha_1 \cosh \alpha_5 \cosh \alpha_n} , \quad (4.18)$$

which is the desired correct value. Having the value of a_H , the flux of the energy-momentum tensor is finally determined as

$$\begin{aligned} a_o &= \frac{1}{4\pi} A_t^2(r_H) + N_t^r(r_+) \\ &= \frac{1}{4\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n)^2 + \frac{\pi}{12} T_H^2 , \end{aligned} \quad (4.19)$$

which matches exactly with that of the Hawking radiation from the black hole as will be shown below.

Up to now, we have obtained the fluxes of electric charges, Eqs. (4.7) and (4.9), and energy-momentum tensor, Eq. (4.19) via the method of anomaly cancellation. It is an interesting and important problem to check that these results coincide with the usual fluxes of Hawking (black body) radiation from the black hole. Although the radiation in the case of bosons should be treated, we simply consider the fermion case in order to avoid the superradiance problem. The Hawking distribution for fermions is given by the Planck distribution at the Hawking temperature with two electric chemical potentials for the charges e_1 and e_K of the fields radiated from the black hole,

$$N_{e_1, e_K}(\omega) = \frac{1}{e^{(\omega - e_1 \Phi_1 - e_K \Phi_K)/T_H} + 1} , \quad (4.20)$$

where $\Phi_1 = \tanh \alpha_1$ and $\Phi_K = \tanh \alpha_n$. By using this, the electric charge fluxes of Hawking radiation, say F_1 and F_K , can be calculated as

$$\begin{aligned} F_1 &= e_1 \int_0^\infty \frac{d\omega}{2\pi} (N_{e_1, e_K}(\omega) - N_{-e_1, -e_K}(\omega)) \\ &= \frac{e_1}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) , \end{aligned} \quad (4.21)$$

$$\begin{aligned} F_K &= e_K \int_0^\infty \frac{d\omega}{2\pi} (N_{e_1, e_K}(\omega) - N_{-e_1, -e_K}(\omega)) \\ &= \frac{e_K}{2\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n) , \end{aligned} \quad (4.22)$$

which exactly match with Eqs. (4.7) and (4.9). As for the energy-momentum flux of Hawking radiation, say F_E , we can obtain

$$\begin{aligned} F_E &= \int_0^\infty \frac{d\omega}{2\pi} (N_{e_1, e_K}(\omega) + N_{-e_1, -e_K}(\omega)) \\ &= \frac{1}{4\pi} (e_1 \tanh \alpha_1 - e_K \tanh \alpha_n)^2 + \frac{\pi}{12} T_H^2 , \end{aligned} \quad (4.23)$$

which also shows the exact coincidence with the flux of Eq. (4.19). These exact matchings imply that, as first realized in [2], the fluxes of Hawking radiation from the black hole we have been considered are capable of canceling the gauge and the gravitational anomalies at the horizon.

5 Discussion

We have applied the method of anomaly cancellation for calculating the Hawking radiation initiated by Robinson-Wilczek to the non-extremal five-dimensional D1-D5 black hole in string theory, and obtained the fluxes of the electric charge flow and the energy-momentum tensor. The resulting fluxes match exactly with those of the two-dimensional black body radiation at the Hawking temperature. The point is that the Hawking radiation plays the role of canceling possible gauge and gravitational anomalies at the horizon to make the gauge and diffeomorphism symmetry manifest at the horizon. This confirms that the anomaly analysis proposed in [2, 4] is still working and valid for a typical black hole in string theory.

What we have considered in the black hole background is the scalar field, which corresponds to a point-like object, that is, point particle. As already mentioned, it cannot have minimal coupling to the two-form gauge field. This gives the basic reason that the two-form gauge field does not enter the story. One possibility for introducing the effect of the two-form gauge field in the two-dimensional action (3.3) is to consider the dual gauge field. Note that the dual of the two-form gauge field in five dimensions is one-form gauge field. So, the field can couple minimally to the dual gauge field, and the nature of the charge carried by the field becomes magnetic from the viewpoint of the original two-form gauge field. It would be interesting to see what one obtains when the dual field is also considered.

The present work is based purely on the viewpoint of quantum field theory, though the black hole we are interested in has the string theory origin. In other words, we have not minded whether the complex scalar field φ is in the field contents of type IIB string theory compactified on five torus. Upon compactification, many moduli fields appear in the low energy supergravity action. Some of them, especially the fixed scalar, are distinguished from the usual scalar field. What we see when such fields are considered instead of the field φ in applying the method of anomaly cancellation may be an interesting question.

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References

- [1] S. W. Hawking, “Particle Creation By Black Holes,” *Commun. Math. Phys.* **43** (1975) 199 [Erratum-ibid. **46** (1976) 206].
- [2] S. P. Robinson and F. Wilczek, “A relationship between Hawking radiation and gravitational anomalies,” *Phys. Rev. Lett.* **95** (2005) 011303 [arXiv:gr-qc/0502074].
- [3] S. M. Christensen and S. A. Fulling, “Trace Anomalies And The Hawking Effect,” *Phys. Rev. D* **15** (1977) 2088.
- [4] S. Iso, H. Umetsu and F. Wilczek, “Hawking radiation from charged black holes via gauge and gravitational anomalies,” *Phys. Rev. Lett.* **96** (2006) 151302 [arXiv:hep-th/0602146].
- [5] S. Iso, H. Umetsu and F. Wilczek, “Anomalies, Hawking radiations and regularity in rotating black holes,” *Phys. Rev. D* **74** (2006) 044017 [arXiv:hep-th/0606018].
- [6] K. Murata and J. Soda, “Hawking radiation from rotating black holes and gravitational anomalies,” *Phys. Rev. D* **74** (2006) 044018 [arXiv:hep-th/0606069].
- [7] E. C. Vagenas and S. Das, “Gravitational anomalies, Hawking radiation, and spherically symmetric black holes,” *JHEP* **0610** (2006) 025 [arXiv:hep-th/0606077].
- [8] M. R. Setare, “Gauge and gravitational anomalies and Hawking radiation of rotating BTZ black holes,” *Eur. Phys. J. C* **49** (2007) 865 [arXiv:hep-th/0608080]; Q. Q. Jiang, S. Q. Wu and X. Cai, “Hawking radiation from the (2+1)-dimensional BTZ black holes,” arXiv:hep-th/0701048.
- [9] Z. Xu and B. Chen, “Hawking radiation from general Kerr-(anti)de Sitter black holes,” *Phys. Rev. D* **75** (2007) 024041 [arXiv:hep-th/0612261].

- [10] S. Iso, T. Morita and H. Umetsu, “Quantum anomalies at horizon and Hawking radiations in Myers-Perry black holes,” arXiv:hep-th/0612286.
- [11] Q. Q. Jiang and S. Q. Wu, “Hawking radiation from rotating black holes in anti-de Sitter spaces via gauge and gravitational anomalies,” Phys. Lett. B **647** (2007) 200 [arXiv:hep-th/0701002].
- [12] Q. Q. Jiang, S. Q. Wu and X. Cai, “Hawking radiation from the dilatonic black holes via anomalies,” Phys. Rev. D **75** (2007) 064029 [arXiv:hep-th/0701235].
- [13] X. Kui, W. Liu and H. Zhang, “Anomalies of the Achucarro-Ortiz black hole,” Phys. Lett. B **647** (2007) 482 [arXiv:hep-th/0702199].
- [14] S. Iso, T. Morita and H. Umetsu, “Higher-spin currents and thermal flux from Hawking radiation,” arXiv:hep-th/0701272.
- [15] G. T. Horowitz, J. M. Maldacena and A. Strominger, “Nonextremal Black Hole Microstates and U-duality,” Phys. Lett. B **383** (1996) 151 [arXiv:hep-th/9603109].
- [16] K. Skenderis, “Black holes and branes in string theory,” Lect. Notes Phys. **541** (2000) 325 [arXiv:hep-th/9901050]; A. W. Peet, “TASI lectures on black holes in string theory,” arXiv:hep-th/0008241; J. R. David, G. Mandal and S. R. Wadia, “Microscopic formulation of black holes in string theory,” Phys. Rept. **369** (2002) 549 [arXiv:hep-th/0203048].
- [17] J. H. Horne, G. T. Horowitz and A. R. Steif, “An Equivalence between momentum and charge in string theory,” Phys. Rev. Lett. **68** (1992) 568 [arXiv:hep-th/9110065].
- [18] G. T. Horowitz and A. Strominger, “Black strings and P-branes,” Nucl. Phys. B **360** (1991) 197.
- [19] G. T. Horowitz and A. Strominger, “Counting States of Near-Extremal Black Holes,” Phys. Rev. Lett. **77** (1996) 2368 [arXiv:hep-th/9602051].
- [20] J. Maharana and J. H. Schwarz, “Noncompact symmetries in string theory,” Nucl. Phys. B **390** (1993) 3 [arXiv:hep-th/9207016]; A. Sen, “Electric magnetic duality in string theory,” Nucl. Phys. B **404** (1993) 109 [arXiv:hep-th/9207053].
- [21] R. A. Bertlmann, *Anomalies in Quantum Field Theory* (Oxford Science Publications, Oxford, 2000).

- [22] W. A. Bardeen and B. Zumino, “Consistent And Covariant Anomalies In Gauge And Gravitational Theories,” Nucl. Phys. B **244** (1984) 421.
- [23] L. Alvarez-Gaume and E. Witten, “Gravitational Anomalies,” Nucl. Phys. B **234** (1984) 269.
- [24] R. A. Bertlmann and E. Kohlprath, “Two-dimensional gravitational anomalies, Schwinger terms and dispersion relations,” Annals Phys. **288** (2001) 137 [arXiv:hep-th/0011067].